Introduction to Tokamak Core Turbulence

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Flux-Gradient Relation



Generalization of Fick's Law:

$$\begin{pmatrix} Q_i \\ Q_e \\ \Gamma \\ \Gamma_{\phi} \end{pmatrix} = - \begin{bmatrix} \chi_i & \cdots & \cdots & \cdots \\ \cdots & \chi_e & \cdots & \cdots \\ \cdots & D & \cdots \\ \cdots & \cdots & D & \cdots \\ \cdots & \cdots & \chi_{\phi} \end{bmatrix} \begin{pmatrix} \nabla T_i \\ \nabla T_e \\ \nabla n \\ \nabla U_{\phi} \end{pmatrix}$$



Properties of Tokamak Core Turbulence

Gyrokinetic Description of Microturbulence

Some Outstanding Confinement Physics Issues: Ion Thermal Transport Electron Thermal Transport Momentum Transport and Transport Barrier

Emphasis: Physics Mechanisms, Recent Progress



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Microinstabilities in Tokamaks

- Tokamak transport is usually anomalous, even in the absence of large scale MHD
- Caused by small scale collective instabilities driven by gradients in temperature, density,...
- Instabilities saturate at low amplitude due to nonlinear mechanisms
- \bullet Particles $\mathbf{E}\times\mathbf{B}$ drift radially due to fluctuating electric field

Amplitude of Tokamak Micro-turbulence



- Relative fluctuation amplitude $\delta n/n_0$ at core typically less than 1 %
- \bullet At the edge, it can be greater than 10 %
- Confirmed in different machines using different diagnostics



 $k_{\theta} \; \rho_i \thicksim 0.1$ - 0.2

-from Mazzucato et al., PRL '82 (μ-wave scattering on ATC) Fonck et al., PRL '93 (BES on TFTR)

-similar results from

TS, ASDEX, JET, JT-60U and DIII-D



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Cinématique Aspect de Gyrocinetique Théorie



GTC simulation of ITG Turbulence: S. Ethier et al.



Electrostatic Microinstabilities in Tokamaks

Classification:	Spatio-temporal Scales	Accessibility Mechanism
Free energy	(wavelength, frequency direction, rough mag.)	for Instability
Trapped Ion Mode n, T _e , (ITG-TIM) T _i	$\sim \rho_{\theta} \sim \omega_{e}^{*}$	Trapped ion precession resonance (coll-less) Collisions btwn trapped and passing ions (dissipative)
Ion Temp. Grad. Mode T _i	> ρ _i < ω* _{pi}	Bad curvature or Negative compressibility
Trapped Electron Mode n or T _e	~ρ _i <ω* _e	Trapped electron precession resonance (coll-less) Collisions btwn trapped and passing e ⁻ s (dissipative)
Electron Temp. G Mode T _e	>ρ _e < ω* _{pe}	Bad curvature or Negative compressibility



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Heuristic Estimation of Diffusion Coefficient

$$\gamma = \gamma_{lin} - k_{\perp}^2 D_{turb} \to 0$$

 Nonlinear coupling induced dissipation leads to saturation (B. Kadomtsev '65)



• $D_{turb} \sim \gamma_{lin}/k_{\perp}^2 \sim (v_{Ti}/a)\rho_i^2 \sim \frac{\rho_i cT_i}{a eB}$: GyroBohm scaling; since $\gamma_{lin} \propto \omega_* \propto (v_{Ti}/a)$, and $k_{\perp} \propto \rho_i^{-1}$.

[Frieman and Chen, Phys. Fluids 1982]

Minimum number of ordering assumptions

- $\frac{\omega}{\Omega_i} \sim \frac{k_{||}}{k_{\perp}} \sim \epsilon_{k,\omega} << 1$; from spatio-temporal scales of fluctuations
- $k_{\perp}\rho_i \sim 1$ for generality: Short wavelength modes (with higher γ_{lin}) can affect the modes at NL peak ($k_{\perp}\rho_i \sim 0.1 \sim 0.2$) through NL coupling. $\rightarrow \omega \sim k_{||}v_{Ti}$ for wave-particle resonance
 - i.e., Landau damping
- $\frac{\delta f}{f_0} \sim \frac{e \delta \phi}{T_e} \sim \frac{1}{k_\perp L_p} \sim \epsilon_{\phi} << 1$; from small relative fluctuation amplitude
- $-k_{\perp} \frac{e\delta\phi}{T_e} \sim \frac{1}{L_p}$: **E** × **B** Nonlinearity ~ Linear Drive
- $\delta n/n_0 \sim \rho/L \sim$ roughly experimental values.

Standard Nonlinear Gyrokinetic Ordering II.

- While the physics origins of $\epsilon_{k,\omega}$ and ϵ_{ϕ} are different, the maximal ordering for NL GK corresponds to $\epsilon_{k,\omega} \sim \epsilon_{\phi}$
- ϵ_{k,ω} >> ϵ_φ leads back to the linear Gyrokinetics: Rutherford-Frieman, Phys. Fluids 11, 569 '68 Taylor-Hastie, Plasma Phys. 10, 419 '68 Tang, Nuclear Fusion 18, 1089 '78 Antonse-Lane, Phys. Fluids 23, 1205 '80 Horton, Rev. Mod. Phys 71, 735 '99
- With $\epsilon_{k,\omega} << \epsilon_{\phi}$, one cannot recover the linear dispersion relation of instabilities. Self-sustained Turbulence, BS by **BDS**

[eg., Frieman and Chen, Phys. Fluids 1982]

- Foundations of Tokamak Nonlinear Kinetic Theory for analytic applications, ballooning codes...
- Number of assumptions minimum
- Based on direct gyro-phase average of Vlasov equation Lots of algebra and book keeping
- Direct expansions in ϵ : Self-consisten up to $O(\epsilon^2) \rightarrow$ Should be fine for linear phase and saturation due to $E \times B$ nonlinearity
- Velocity space nonlinearity: $\nabla_{\parallel} \delta \phi \partial_{v\parallel} \delta f \sim O(\epsilon^3)$ is ignored. Energy, phase space volume **not** conserved.
- May not be able to describe long term behavior accurately Topic of Current Research: [Villard, Hatzky, Sorge, Lee, Wang]

 \rightarrow Physics responsible for the difference?

Conventional Nonlinear GK Derivation: Heuristic

• Transforming to guiding center variables, $\mathbf{R} = \mathbf{x} + \rho$ $\mu = v_{\perp}^2/2B, \mathbf{v} = v_{\parallel}\mathbf{b} + v_{\perp}(\mathbf{e}_1 cos\theta + \mathbf{e}_2 sin\theta)$ one can write Vlasov equation as

$$\frac{\partial}{\partial t}f + v_{\parallel}\mathbf{b}\cdot\frac{\partial}{\partial \mathbf{R}}f + \frac{\mathbf{E}\mathbf{x}\mathbf{b}}{B}\cdot\frac{\partial}{\partial \mathbf{R}}f + (q/m)E_{\parallel}\frac{\partial}{\partial v_{\parallel}}f - \Omega\frac{\partial}{\partial\theta}f = 0$$

- Since $\Omega >> \omega$, to the lowest order $\Omega \frac{\partial}{\partial \theta} f = 0$
- Writing $f = \langle f \rangle + \tilde{f}$, with $f = \langle f \rangle >> \tilde{f}$ in which $\langle ... \rangle$ a gyro-phase average,

$$\frac{\partial}{\partial t}\langle f\rangle + v_{\parallel}\mathbf{b}\cdot\frac{\partial}{\partial \mathbf{R}}\langle f\rangle + \frac{\mathbf{Exb}}{B}\frac{\partial}{\partial \mathbf{R}}\langle f\rangle + (q/m)E_{\parallel}\frac{\partial}{\partial v_{\parallel}}\langle f\rangle - \Omega\frac{\partial}{\partial\theta}\tilde{f} = 0,$$

which is a solubility condition for $\langle f \rangle$.

- Gyro-phase averaging, one gets an electrostatic NL GK in a uniform B field: $\frac{\partial}{\partial t}\langle f \rangle + v_{||} \mathbf{b} \cdot \frac{\partial}{\partial \mathbf{R}} \langle f \rangle + \frac{\langle \mathbf{E} \rangle \mathbf{x} \mathbf{b}}{B} \frac{\partial}{\partial \mathbf{R}} \langle f \rangle + (q/m) \langle E_{||} \rangle \frac{\partial}{\partial v_{||}} \langle f \rangle = 0$
- Frequency-wave number expansion and amplitude expansion, and geometric expansion (if it were included) are all lumped together in this procedure. If one modifies an ordering, needs to do the derivation all over again

Nonlinear Gyrokinetics for Large Scale Computation

- Direct simulation of actual size fusion plasmas in realistic geometry using the primitive nonlinear plasma equations (Vlasov-Maxwell), is far beyond the computational capability of foreseeable future.
- For turbulence problems in fusion plasmas, the temporal scales fluctuations much longer than the period of a charged particle's cyclotron motion, while the spatial scales and gyro-orbits are much smaller than the macroscopic length scales: → details of the charged particle's gyration motion are not of physical interest → Develop reduced dynamical equations which capture the essential features
- After decoupling of gyro-motion, gyrokinetic equation describes evolution of gyro-center distribution function, independent of the gyrophase, θ , defined over a five-dimensional phase space $(\mathbf{R}, v_{\parallel}, \mu)$. \rightarrow save enormous amounts of computing time by having a time step greater than the gyro-period, and by reducing the number of dynamical variables.
- In gyrokinetic approach, gyro-phase is an ignorable coordinate, magnitude of the perpendicular velocity enters as a parameter in terms of an adiabatic invariant μ
- Nonlinear gyrokinetic equations are now widely used in turbulence simulations.

Modern Nonlinear Gyrokinetics

- Starting from the original Vlasov-Maxwell system (6D), pursue "Reduction of dimensionality" for both computational and analytic feasibility.
- Keep intact the underlying symmetry/conservation of the original system.
- Perturbation analysis consists of near-identity coordinate transformation which "decouples" the gyration from the slower dynamics of interest in the single particle Lagrangian, rather than a direct "gyro-phase average" of Vlasov equation.
- This procedure is reversible:

The gyro-phase dependent information can be recovered when it is needed.

Proto-type Modern Nonlinear Gyrokinetics Hamiltonian Derivation

[since Dubin *et al.*, PF '88, followed by Hagan-Frieman, Yang-Choi,...]

- Recognized the importance of Hamiltonian structure, simplicity of derivation
- Unnecessary to use canonical variables with mixed variable generating functions
- Essence of Hamiltonian perturbation theory is *symplectic structure* (Littlejohn, J. Math. Phys. **20**, 2445 (1979)).
- Using Darboux Theorem, Dubin *et al.*, derived an energy conserving set of nonlinear gyrokinetic equations in uniform magnetic field
- Extended to general geometry by Hagan-Frieman (1985) and by Yang-Choi (1985), energy conservation *not* achieved.

Phase Space Lagrangian Derivation of Nonlinear Gyrokinetics

[since Hahm, PF **31**, 2670 '88, followed by Brizard, Sugama,...]

- Conservations Laws are Satisfied.
- Various expansion parameters appear at different stages
 → Flexibility in variations of ordering
 for specific application
- Guiding center drift calculations in equilibrium field B: Expansion in $\delta_B \equiv \rho_i / L_B \sim \rho_i / R$.
- Perturbative analysis consists of near-identity transformations to new variables which remove the gyro-phase dependence in perturbed fields $\delta A(\mathbf{x}), \delta \phi(\mathbf{x})$ where $\mathbf{x} = \mathbf{R} + \rho$: Expansion in $\epsilon_{\phi} \equiv e(\delta \phi - \frac{v_{||}}{c} \delta A_{||})/T_e \sim \delta B_{||}/B_0$
- Derivation more transparent, less amount of algebra

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Ion Thermal Transport

- * Better understood, compared to other transport channels
- * In Aux-heated plasmas, typically $\chi_i >> \chi_{i,Neo}$ (cf. ITBs)
- * Ion Temperature Gradient (ITG) Turbulence: Best Candidate
- With recent advances in gyrokinetic codes, simulation results begin to converge for simple cases, not only in numbers, but also in underlying physics.

The effective upshift of onset condition for large ion heat flux is caused by Zonal flows

[Dimits et al., Phys. Plasmas, 2000] from Cyclone project





What is a zonal flow?



Basic Physics of a Zonal Flow

from Diamond, Itoh, Itoh, and Hahm, "Zonal Flows in Plasma-a Review" PPCF '05



Damping by Collisions

Spectrum peak at GAMs, in addition to zero freq., are observed worldwide



See also other tokamaks (e.g., JIPP-TIIU, HT-7,....) from APS-Review 2006, by K. Itoh

Key Physics Mechanisms behind Size Scaling



q dependence of transport from Zonal Flow behavior

I_p scaling of confinement:
 one of the remaining puzzles of ion thermal transport
 Zonal flow characteristics depend on q values:

GAMs can exist only in high q region. In low q region, Stationary Zonal Flows persists.

GAMs are less effective in reducing turbulence and transport than Stationary ZFs, due to its high frequency [Hahm et al., PoP '99]

Transport is consequently lower for lower q value from gyrofluid simulations [Miyato et al., IAEA '04]

Similar results from GK simulations [Angelino et al., PPCF '06]

Experimental Relevance?

q profiles



GAM

Stationary Zonal Flows





Collisionality dependence of Ion Thermal Transport

Near ITG marginality, collisional damping of zonal flow controls the transport.

[Lin, Hahm, Lee, Tang, and Diamond, PRL '99]

Transport increases with Ion Collisionality.

On the other hand, trapped electron response decreases with collisionality.

[D. Mikkelsen et al, IAEA '02, paper EX-P5-03]

Transport decreases with e⁻ collisionality.

While some experiments [DIII-D, C-mod, JET,...] report transport increase with collisionality, further parameter scans are required to identify the underlying physics mechanisms



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Electron Thermal Transport is Anomalous

- ALL operational modes of tokamaks and STs have exhibited anomalous ٠ electron thermal transport.
- There is evidence that electron temperature profiles are stiff ٠ in tokamaks:

Perturbative Experiments (e.g., Heat Pulse Propagation)

- Apparent Heat Pinch in aux-heated plasmas
- Possible theoretical candidates include: •
 - Trapped Electron Mode (and ITG with trapped electrons)
 - Electron Temperature Gradient Mode
 - Magnetic Flutter
 - Something other than Turbulence



Trapped Electron Modes

- Can produce χ_e of experimental relevance when strongly turbulent.
- Electron heat transport in ASDEX-U ECH plasmas exhibits:

Threshold behavior in agreement with TEM theory

Dependence on R/L_{Te} ~ that of γ_{lin} [F. Ryter, PRL '05]



In JET, low-k fluctuation reduction accompanies electron thermal transport barrier formation via localized electron heating [G. Conway., PPCF '02]

From X-mode Reflectometry Fluctuation propagates in ω_{*e} direction $\lambda_{\perp} \sim 11$ cm

TEM ?



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Radial Correlation Length of ETG Turbulence can exceed ρ_{e}

- To produce electron transport at the level of experimental relevance, radial correlation length has to be significantly enhanced over ρ_{e}
- Radially elongated streamers are observed from many simulations, but there's no consensus on "enhancement factor of transport " as yet





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$\chi_e\,$ anomalous, even when low-k fluctuations are stable

Predominant Electron Heating via Energetic Particles in NSTX --- ITER relevant





to look for ETG streamers with good spatial resolution

Stutman

104

0.2 0.3

0.4

r/a

0.5

0.6

0.7

Mazzucato, Park





Reducing χ_e in Burning Plasmas

• Various transport channels behave differently in forming ITBs.

• In most cases, Electron heat ITBs are formed by localized electron heating on RS plasmas [cf., JT-60U high triangularity, PS plasmas]

 The following stabilization mechanisms are likely to be ineffective in BP: Density peaking for ITG, ETG,... High T_e/T_i for ETG (High T_i/T_e for ITG) NBI-driven Flow Shear (?)

• q - profile control (eg., RS) remains effective in reducing electron heat transport in Burning Plasmas:

Precession reversal of trapped particles Suppression of ETG streamers Stabilization of NTM



Performance and Stability of RS plasmas

• BOX-type profiles and radially localized reduction of χ_e in tokamak RS plasmas (JT-60U, DIII-D,...): not ideal for performance and macroscopic stability





JT-60U

10.0

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Momentum Transport is important in ITER

 Plasma Rotation beneficial, not only for ExB shear suppression of turbulence, but also for stabilization of Resistive Wall Modes.

• Will ITER have enough Wave and Neutral Beam power

to generate sufficient rotation?

Empirical scaling [Rice, NF 2001] with stored energy and I_p works.

But size scaling unknown:

extrapolation to ITER?

Inter-machine comparisons underway.

Ince-Cushman & the ITPA Transport Physics Group





B*-symmetry breaking unifies two mechanisms:

from $m_i B^* dv_{\parallel} / dt = -(eB + m_i cv_{\parallel} \nabla \times \hat{b}) \cdot \nabla \delta \phi$ [Hahm,NLGK, PF '88]

	Gurcan, Diamond, Hahm, Singh [Phys. Plasmas 14, 042306 '07]	Hahm, Diamond, Gurcan, Rewoldt [Phys. Plasmas, July '07]
Net acceleration of parallel flow:	$-e_{i}B abla_{\parallel}\delta\phi$	$-m_i c \mathbf{v}_{\parallel} \nabla \times \hat{b} \cdot \nabla \delta \phi$
Symmetry-breaking:	k over the spectral width	curvature drift $\sim \hat{b} \times (\hat{b} \cdot \nabla)\hat{b}$ over the flux surface
Provided by:	mean E x B shear shifting fluctuations radially	ballooning mode structure causing finite net parallel acceleration over the flux surface
Main consequence:	residual stress driven by $\mathbf{E} \mathbf{x} \mathbf{B}$ shear (or $\nabla P_i / n_i$ and velocity shear via radial force balance)	convective pinch-like term (the TEP-like piece is insensitive to mode details)
Most likely to be relevant for:	plasmas with strong E x B shear, incl. H-mode, ITB's	pinch is likely to be inward for OH and electron-heated plasmas 2



Intrinsic Rotation: A particle-quasiparticle approach



[1] Gürcan Ö.D., Diamond P.H., Hahm T.S., et.al. Phys Plasmas, April, 2007



Poloidal Flows deviates from Neoclassical Prediction

Formation of ITBs (TFTR, JET) and L-H Transition (DIII-D) often initiate with a sharp change in U_{θ} which is different from neoclassical theory prediction.

Other possibilities include zonal flows driven by turbulence via poloidal Reynolds stress: $\Pi_{r\theta} = \langle V_r U_{\theta} \rangle$: Topic of active research in TH and EXPT, in the last decade. JET [Crombe et al., PRL '05], DIII-D [Solomon et al., PoP '06]





H-mode Transition



We still lack quantitative predictions on: Trigger, Power Threshold, Pedestal Width,... ITER: tight margin on P_{LH} Hysteresis may help (eg., H-mode entry at low n,...) but local prediction too optimistic ? [Lebedev-PHD '98]

Paradigm Extension:

S-curve at fixed r ---> Flux Gradient Landscape [Diamond et al., PRL '97] Deterministic Theory ---> Probabilistic Theory L-H transition probability [Itoh, Itoh, Fukuyama, Yagi]







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Remaining Issues

- Pedestal Physics: no clear trend from empirical scalings
- L-H Power Threshold and Transition Physics: not much theory insight
- Edge Localized Modes: beyond mode identification
- Particle Pinch: quasi-linear prediction depends on collisionality, η_e , ... equi-partition theory fits exps better (cf. RS plasmas)?
- Gyrokinetic Simulation of Momentum Transport
- Transport Control via Flow Shear (IBW in upcoming Asian tokamaks?)
- Interactions among Different Topical Areas:

. . . .

TAE, RS-AE,...: Role in ITB Physics?

Turbulence Effects on NTM : Nonlinear Threshold?



Rotation Plays a Central Role in Magnetic Confinement

